This article presents experimental research results data and optimization of a two-stage cyclone-type dust concentrator and the design and operating parameters data. To resolve the assigned optimization task, SUMT (Sequential Unconstrained Minimization Technique) was applied. The Davidon-Fletcher-Powell method was used to find the minimum of a function, afterwards the iterative procedure was realized. The results were processed using joint analysis of the characteristic curve with regression equation of dust collecting efficiency and liquid resistance.

**1 Introduction**

A large part of the technological processes of building materials enterprises, as well as a number of other branches of industry are the mechanical manipulation and material handling processes, related to the gravity flow of large amounts of granular materials on closed gutters and accompanied by intensive release of dust.

Traditional aspiration systems, characterized by volumetric emission of dusty air, are widely used for localization of sources of dust emissions. Reduction of the power consumption and the dust emission of the system can be achieved as a result of purification of the aspirated air in centrifugal dust concentrators – cyclones, the conclusion of the trapped dust of which is carried out with part of the carrier gas stream [1].

The working principle of the two-stage cyclone-type dust concentrator is ejection of particles from the gas-dust flow by centrifugal force (Figure 1). Through the inlet pipe into the cyclone part of the dust concentrator between the body and the cylindrical insert a polluted gas and dust stream is fed. Particles of dust by centrifugal force hit the walls of the body and are carried out from the internal cavity of the two-stage dust concentrator by the air flow through the annular cavity and then enter the recirculation line. Part of the air flow, which remained after the first stage of purification, is di-
rected to the countercurrent cyclone for the second stage of purification through the annular gap between the upper end of the cylindrical insert and the lid.

2 Methodology of determination of the two-stage cyclone-type dust concentrator optimal design parameters

As a result of the experimental research, it was found that the flow rate of the air supplied for cleaning exerts a global influence on the main part of the efficiency of the two-stage dust concentrator capture and the value of \( \eta_{\text{TSC}} \) increases with the growth in the flow rate of the filtered air due to the first and second stages of purification. Increase in dust collection efficiency of the first \( h_1 \) and second \( h_2 \) stages of purification with a growth in the flow rate of the gas supplied for purification have a positive impact on the dust separation process [2, 3].

Growth of the factor of the relative consumption of recirculation air in the first stage of cleaning \( h_1 \) increases the efficiency of air purification, the cleaning efficiency is reduced due to the decrease in the velocity of the gas flow in the second stage \( h_2 \). Increase the geometric characteristics of the height of the entrance pit in the second stage and the depth of immersion of an exhaust pipe can induce a random decrease or increase in the overall efficiency of the two-stage dust concentrator. Growth of the flow rate of the filtered gas increases the hydraulic resistance of the two-stage dust concentrator in the first stage and lowers in the second stage of purification. Therefore, the use of factors \( X_1, X_2 \) and \( X_3 \) limits the overall efficiency of the two-stage dust concentrator [4].

Optimization of the two-stage dust concentrator design and operating parameters was reduced to the determination of the values of the factors entering into the regression equation (1), when the maximum value of the response function is achieved – the efficiency of dust collection of the two-stage dust concentrator [5].

\[
\begin{align*}
\eta_{\text{TSC}} &= 91.15 - 0.18 \cdot X_1 + 0.26 \cdot X_2 + 0.29 \cdot X_3 + 0.22 \cdot X_4 + 0.03 \cdot X_5 \\
&\quad - 0.88 \cdot X_6 \cdot X_7 - 0.79 \cdot X_8 - 0.5 \cdot X_9 - 0.44 \cdot X_{10} - 0.13 \cdot X_{11} \quad (1)
\end{align*}
\]

However, the maximum possible efficiency of dust collection of the two-stage dust concentrator can take place at the values of the factors \( X_1, X_2, X_3, X_4 \), which are unacceptable for the operation conditions for the following reasons:

1. Optimal relative recirculation airflow rate \( X_1 \) in most cases, in practice, will not correspond to the required flow rate in the recirculation line of the aspiration system.
2. Optimal values of factors may be unacceptable, because they can create the unacceptably high hydraulic resistance of the machine. This, above all, refers to the flow of filtered air \( (X_1) \), which exerts a dominant influence both on the cleaning efficiency \( (1) \), and on the hydraulic resistance \( (2) \) of the two-stage dust concentrator:

\[
\Delta P_{\text{TSC}} = 1605 - 64 \cdot X_1 + 66 \cdot X_2 + 532 \cdot X_3 + 71 \cdot X_4 + 69 \cdot X_5 + 62 \cdot X_6 + 78 \cdot X_7 \quad (2)
\]

Therefore, in the future factor \( X_4 \) was excluded from consideration and fixed at certain levels of variation. And the optimal values of the factors \( X_1, X_2, X_3 \) were limited to the maximum possible level of the hydraulic resistance of the machine [4].

3 Optimization task solution

To resolve the assigned optimization task, SUMT (Sequential Unconstrained Minimization Technique) was applied firstly proposed by Carroll [6] and which has been developed further by Fiacco and McCormick [7, 8], which is based on the idea of converting the task of minimizing a function:

\[
z = f(\bar{x})
\]

with the appropriate restrictions imposed on \( x \), in the problem of finding the minimum without constraints of the function:

\[
Z = f(\bar{x}) + P(\bar{x}) \ldots
\]

The function \( P(x) \) is a penalty function. It is necessary that, in violation of restrictions, it “fine” the function \( Z \), increasing its value. In this case, the minimum of the function \( Z \) will be within the constraint area.

The penalty function \( P(x) \) is written as follows:

\[
P(x) = r \sum_{j=1}^{m} \frac{1}{C_j(\bar{x})}
\]

where \( C_j(x) > 0 \) – the constraints imposed on \( x \), \( m \) – number of constraints, \( r \) – the positive value.

If the imposed constraints are of the form \( h(x) \leq 0 \), then they are written as \( -h(x) \geq 0 \).

With allowance for (5), the function \( Z = \varphi(\bar{x}, r) \) takes the form:

\[
Z = \varphi(\bar{x}, r) = f(\bar{x}) + r \sum_{j=1}^{m} \frac{1}{C_j(\bar{x})}
\]

If \( x \) takes valid values for which \( C_j(\bar{x}) \geq 0 \), then \( Z \) takes values that are greater than the corresponding values of \( f(\bar{x}) \), and the difference can be reduced by the fact that \( r \) can be a very small value. But if \( x \) takes a value that, while acceptable, is close to the...
Table 1 The range of the factor space of the central composite rotatable plan of the four factor experiment

<table>
<thead>
<tr>
<th>Code</th>
<th>Factors</th>
<th>Step</th>
<th>Variation levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Height of the entrance pit in the II stage [mm]</td>
<td>50</td>
<td>-2</td>
</tr>
<tr>
<td>X2</td>
<td>Depth of immersion of an exhaust pipe [mm]</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>X3</td>
<td>Flow of filtered air [m³/h]</td>
<td>100</td>
<td>700</td>
</tr>
<tr>
<td>X4</td>
<td>Relative recirculation airflow rate</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Therefore, as a function f(x) a function

\[ f(\vec{x}) = -\eta_{TSDC} = -f(X_1; X_2; X_3; X_4) \]  
(7)

was adopted, with the factor X4 being fixed at certain levels (-1, 0, +1).

The penalty function with eight imposed constraints

\[ P(x) = r \left( \frac{1}{x_1^{1+1}} + \frac{1}{1-x_1} + \frac{1}{x_2^{1+1}} + \frac{1}{x_2-1} + \frac{1}{x_3^{1+1}} + \frac{1}{x_3-1} + \frac{1}{x_4^{1+1}} + \frac{1}{\Delta P_{TSDC} - A} \right) \]  
(8)

where \( \Delta P_{TSDC} \) – the hydraulic resistance of the two-stage dust concentrator, calculated by the expression (2);

A and B – respectively, the lower and upper levels of the set hydraulic resistance of the machine.

The sum of the expressions (7) and (8) is the initial equation \( \varphi(\vec{x}, r) \), the solution of the minimization task of which makes it possible to obtain the optimal values of the factors \( X_1, X_2, X_3 \) for a given value \( X_4 \) and is performed as follows.

Given the initial point \( \vec{x}_0 \) defined by the values \( X_1, X_2, X_3, X_4 \), and by checking for its admissibility, we form the function \( \varphi(\vec{x}, r) \), assuming that \( r=r_0=1 \).

After finding the minimum of the function \( \varphi(\vec{x}, r_0) \) at the point \( \vec{x}_0 \) using the Davidon-Fletcher-Powell method [7], a new function \( \varphi(\vec{x}, r) \) is formed for which \( r=r/10 \), and \( \vec{x} = \vec{x}_0^{+} \). After that a repeated operation of minimization follows.

Thus, an iterative procedure is performed. At the k step, the function \( \varphi(\vec{x}, r_k) \) is minimized, the minimum of which is at the point \( \vec{x}_k^{+} \) and is used as a subsequent point of the iterative procedure for minimizing the function \( \varphi(\vec{x}, r_{k+1}) \), where \( r_{k+1} = r_k/10 \).

The sequence \( r_k \) decreases and tends to zero, hence the sequence of points of minima converges to the solution of the task of minimizing the function (7).

Proceeding from the above, a program was developed for calculating the optimal values of the factors \( X_1, X_2, X_3, X_4 \) of the regression equation (1) the efficiency of the dust collecting of the two-stage dust concentrator.

The results of calculating the optimal values of the factors \( X_1, X_2, X_3, X_4 \) for a number of fixed val-
ues of the relative recirculation airflow rate ($X_4$) and various limitations of the maximum possible hydraulic resistance of the two-stage dust concentrator are presented in the Table 2.

Analysis of the presented calculation results shows that for each value of the maximum possible hydraulic resistance of the machine and the level of variation in the relative recirculation airflow rate, there is a well-defined set of optimal values of factors $X_1$, $X_2$, $X_3$. Moreover, the values of these factors vary in a rather wide range.

Consequently, the optimum values of the height of the entrance pit in the second stage ($X_1$) and the depth of immersion of an exhaust pipe ($X_2$) of the two-stage dust concentrator will depend on the specified operational parameters of the machine: hydraulic resistance $\Delta P_{\text{TSDC}}$, flow of filtered air ($X_3$) and relative recirculation airflow rate ($X_4$). This greatly complicates the technique of engineering calculation of the proposed machine. Therefore in the future an attempt will be made to fix these factors at certain levels with the exception of factor $X_4$, because its value will be equal to the required flow rate in the recirculation line of the aspiration system [9].

### 4 Implications

Analysis of the dependence of the maximum possible dust collection efficiency of the TSDC on the set level of hydraulic resistance (Figure 3) [10] showed that the optimal value of the hydraulic resistance of the TSDC is 1800...1900 Pa.

Given a level of $\Delta P_{\text{TSDC}} = 1800$ Pa, combinations of the optimal values of factors $X_1$, $X_2$, $X_3$ were determined with $X_4$ varying from -1 to +1 in steps of 0.1 (Figure 4).

### Table 2 The results of calculating the optimal values of factors at a given level of the hydraulic resistance of the two-stage dust concentrator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Level of the hydraulic resistance of the TSDC [Pa]</th>
<th>Level of $X_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1600</td>
<td>1700</td>
</tr>
<tr>
<td>Optimal factor values</td>
<td>$X_1$</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>$X_2$</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>$X_3$</td>
<td>-0.12</td>
</tr>
<tr>
<td>Efficiency of the TSDC [%]</td>
<td>94.23</td>
<td>94.51</td>
</tr>
<tr>
<td>Optimal factor values</td>
<td>$X_1$</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>$X_2$</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td>$X_3$</td>
<td>0.022</td>
</tr>
<tr>
<td>Efficiency of the TSDC [%]</td>
<td>95.24</td>
<td>95.33</td>
</tr>
<tr>
<td>Optimal factor values</td>
<td>$X_1$</td>
<td>-0.23</td>
</tr>
<tr>
<td></td>
<td>$X_2$</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td>$X_3$</td>
<td>0.08</td>
</tr>
<tr>
<td>Efficiency of the TSDC [%]</td>
<td>95.39</td>
<td>95.56</td>
</tr>
</tbody>
</table>

### Table 3 Operational characteristics of the TSDC under optimal and accepted combinations of factor values $X_1$, $X_2$, $X_3$

<table>
<thead>
<tr>
<th>Factors $X_1$, $X_2$, $X_3$</th>
<th>Optimal</th>
<th>Accepted</th>
<th>Optimal</th>
<th>Accepted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency of dust collection [%]</td>
<td>94.7</td>
<td>95.26</td>
<td>1800</td>
<td>1901</td>
</tr>
<tr>
<td>Hydraulic resistance [Pa]</td>
<td>0</td>
<td>95.49</td>
<td>95.46</td>
<td>1800</td>
</tr>
<tr>
<td>Level of variation of the relative recirculation airflow rate ($X_4$)</td>
<td>1800</td>
<td>1759</td>
<td>0</td>
<td>1759</td>
</tr>
</tbody>
</table>

3 Dependence of the efficiency of dust collection of the TSDC from the hydraulic resistance at optimal values of $X_1$, $X_2$, $X_3$. 

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Table 4 The ratio of the basic geometric dimensions (in fractions of $D_k$) of the TSDC

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Body diameter</td>
<td>$D_k$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Diameter of the cylindrical insert</td>
<td>$D_i$</td>
<td>0.63</td>
</tr>
<tr>
<td>3</td>
<td>Diameter of an exhaust pipe</td>
<td>$D_{ext}$</td>
<td>0.31</td>
</tr>
<tr>
<td>4</td>
<td>Internal diameter of annular pit</td>
<td>$D_i$</td>
<td>0.85</td>
</tr>
<tr>
<td>5</td>
<td>Diameter of dust vantage</td>
<td>$d_i$</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>Height of body</td>
<td>$h_i$</td>
<td>1.9</td>
</tr>
<tr>
<td>7</td>
<td>Height of the entrance pit in the II stage</td>
<td>$h_{ent}$</td>
<td>0.33</td>
</tr>
<tr>
<td>8</td>
<td>Height of the cylindrical part of II stage</td>
<td>$h_s$</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>Depth of immersion of an exhaust pipe</td>
<td>$h_{ent}$</td>
<td>0.27</td>
</tr>
<tr>
<td>10</td>
<td>Height of the hopper</td>
<td>$h_{out}$</td>
<td>0.6...0.7</td>
</tr>
<tr>
<td>11</td>
<td>Length of body</td>
<td>$l_i$</td>
<td>1.4</td>
</tr>
</tbody>
</table>

After a joint analysis of the obtained graphical dependencies with the regression equations of the dust collection efficiency (1) and hydraulic resistance (2) of the TSDC, the following combination of factor values was adopted: $X_1 = 0.15; X_2 = 0.25; X_3 = 0.4$.

The regression equations (1) and (2) in this case take the following form:

$$\eta_{TSDC} = 95.46 + 0.07 \cdot X_4 - 0.13 \cdot X_4^2$$  \hspace{1cm} (9)

$$\Delta P_{TSDC} = 1830 - 71 \cdot X_4$$  \hspace{1cm} (10)

5 Conclusions
The data presented in Table 3 show that the decision made allows a significantly increase in the efficiency of dust collection in the area of low recirculation airflow rate, in addition, the efficiency of the TSDC becomes more stable and depends less on $X_4$, which is confirmed by the obtained dependence (9). The increase in hydraulic resistance is insignificant (5.5%), and in the area of high values of the relative recirculation airflow rate there is even a drop in the hydraulic resistance of the machine, which explains a small decrease in the overall efficiency of dust collection.

Taking into account the step and the variation levels (Table 1), the optimal values of the factors after decoding were: the height of the entrance pit in the second cleaning stage ($X_4$) – $h_{ent} = 133$ mm, the depth of immersion of an exhaust pipe ($X_2$) – $h_{out} = 108$ mm, the airflow rate for cleaning ($X_1$) – $Q_a = 940$ m$^3$/hours.

The ratio of the basic geometric dimensions of the optimized design of a two-stage cyclone-type dust concentrator with a body diameter $D_k$ is shown in Table 4. The optimal air velocity in the section of the TSDC was determined by the formula:

$$V_{opt} = \frac{(2 - q) \cdot Q_a}{0.785 \cdot D_k^2}$$  \hspace{1cm} (11)

where $q$ – relative recirculation airflow rate.

Considering the insignificant effect of the relative recirculation airflow rate on the cleaning efficiency of the two-stage dust concentrator, the optimal fictitious gas velocity in the section of the machine was calculated for $q = 0.4$ and amounted to 3.33 m/s [4].

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REFERENCES