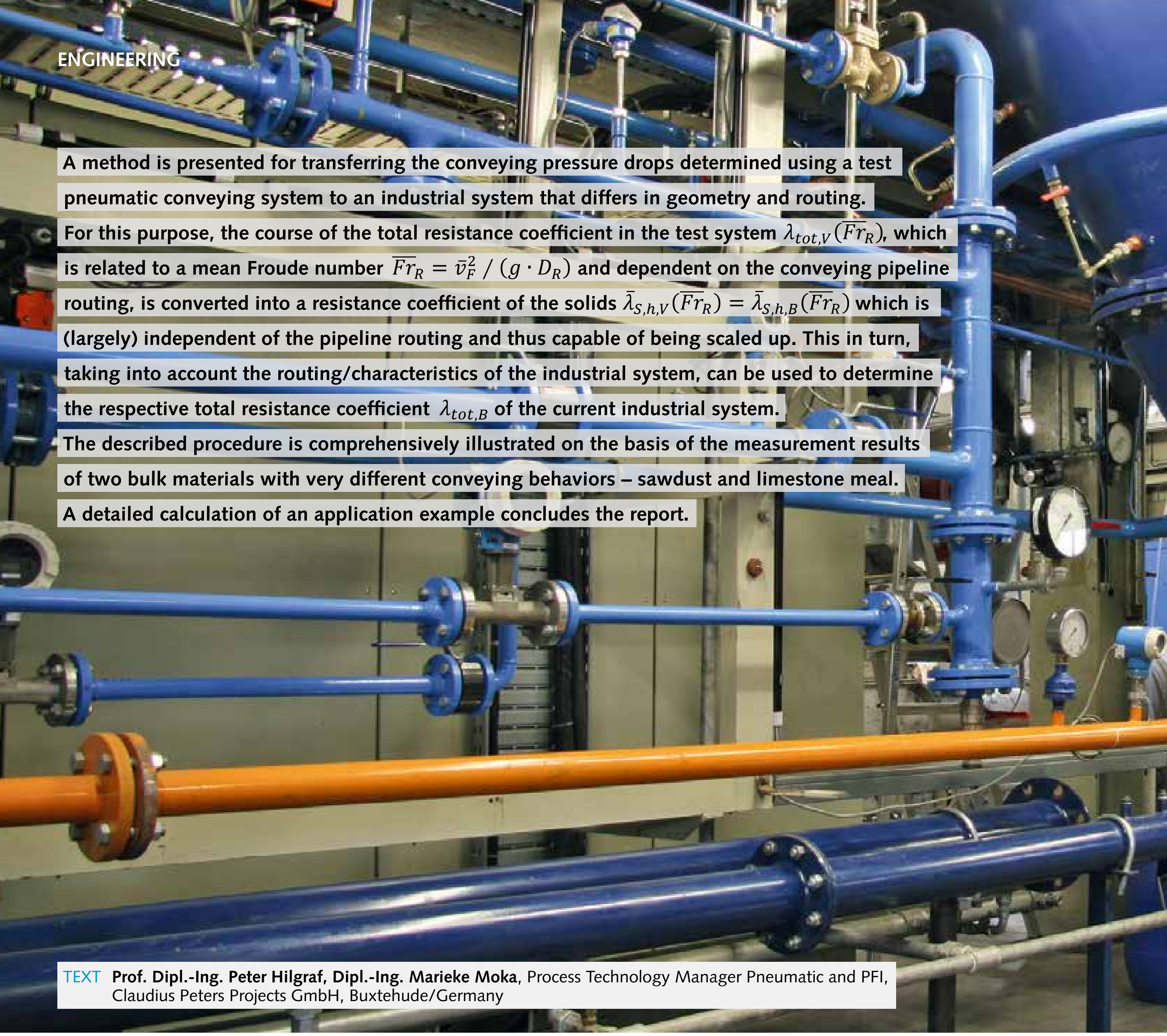


A method is presented for transferring the conveying pressure drops determined using a test pneumatic conveying system to an industrial system that differs in geometry and routing. For this purpose, the course of the total resistance coefficient in the test system $\lambda_{tot,V}(\overline{Fr}_R)$, which is related to a mean Froude number $\overline{Fr}_R = \bar{v}_F^2 \,/\, (g \cdot D_R)$ and dependent on the conveying pipeline routing, is converted into a resistance coefficient of the solids $\bar{\lambda}_{S,h,V}(\overline{Fr}_R) = \bar{\lambda}_{S,h,B}(\overline{Fr}_R)$ which is (largely) independent of the pipeline routing and thus capable of being scaled up. This in turn, taking into account the routing/characteristics of the industrial system, can be used to determine the respective total resistance coefficient $\lambda_{tot,B}$ of the current industrial system. The described procedure is comprehensively illustrated on the basis of the measurement results of two bulk materials with very different conveying behaviors – sawdust and limestone meal. A detailed calculation of an application example concludes the report.

TEXT **Prof. Dipl.-Ing. Peter Hilgraf, Dipl.-Ing. Marieke Moka**, Process Technology Manager Pneumatic and PFI, Claudius Peters Projects GmbH, Buxtehude/Germany

CLAUDIUS PETERS PROJECTS GMBH

Direct scaling up of pneumatic conveying tests to industrial systems

1 Introduction

When conducting orientational conveying trials in a test system pipeline (L_R, D_R) to assess the suitability in principle of a new/not yet pneumatically conveyed bulk material, usually only the mass flows of solids $\dot{M}_S$ and conveying gas $\dot{M}_F$, as well as the total pressure drop $|\Delta p_R|$ at a given back pressure p_{out} are measured as a function of the respective operating conditions at the start and end of the pipeline, i.e. the pressures (p_{in}, p_{out}), the gas velocities $(v_{F,in}, v_{F,out})$ and the loading $\mu = \dot{M}_S \,/\, \dot{M}_F$, with: $\dot{M}_S, \dot{M}_F$ = solids and gas mass flow, are recorded by measurement. The same conditions are usually also present in industrial systems in which the operator implements different load cases, e.g. by varying the above-mentioned parameters. From the measured variables determined in this way, a total resistance coefficient λ_{tot} of the current conveying situation can be calculated via the adapted pressure drop equation

$$|\Delta p_R| = \lambda_{tot} \cdot \mu \cdot \frac{L_R}{D_R} \cdot \frac{\varrho_F}{2} \cdot \bar{v}_F^2 \qquad (1)$$

In this equation, $\bar{v}_F$ is a suitably defined mean

All Claudius Peters/Authors

Test stand in the Claudius Peters technical center for pneumatic conveying

gas velocity, while $\bar{\varrho}_F$ is the corresponding mean conveying gas density. Both will be discussed in more detail below. The λ_{tot}-values of the operating conditions investigated in the test pipeline describe their respective mean pressure drop resistance and summarize the individual resistances of the given conveying pipeline, i.e. they are pipeline-specific and cannot be used directly for the design of different conveying pipelines.

In the following, a method is presented for transferring these "summary" test results to operating plants which differ geometrically from the test pipeline, e.g. with regard to pipeline length L_R, pipe inside diameter D_R and/or pipeline route. The quality of the transfer is checked with the aid of measurement results for the bulk materials sawdust and limestone meal.

The following stipulations apply here:

» True velocities are designated by "u_X" and so-called empty pipe velocities by "v_X". For the gas phase in pneumatic conveying pipelines, for instance $v_F = \varepsilon_F \cdot u_F$ then applies, with $\varepsilon_F =$ as relative gap volume of the gas. Because ε_F in such systems has values of $\varepsilon_F \gtrsim 0.90$, it is generally possible to assume $u_F = v_F$ with justifiable accuracy. In practice, this assumption is generally applied in the evaluation of tests.

» Differences $\Delta\Psi$ or differentials $d\Psi$ of a variable Ψ are defined, as is usual in mathematics, as differences of the quantity Ψ_{out} leaving the piping-/calculation section under consideration minus the quantity Ψ_{in} entering it, i.e. $(\Delta\Psi, d\Psi) = (\Psi_{out} - \Psi_{in})$. Pipeline pressure drops have a negative sign because of $(p_{out} < p_{in})$, while e.g. the pressure increase of a compressor $(p_{out} > p_{in})$ has a positive sign.

2 Pressure drop calculation of pneumatic conveying pipelines

The following section provides an introduction to the equations required for calculating the pressure drop of pneumatic conveying systems. This is followed by a description of the procedure commonly used in practice to determine this total pressure drop.

The total pressure drop of a conveying system consisting of horizontal and vertical pipe sections as well as pipe bends, switches, etc. is the sum of the components

$$\begin{aligned}\Delta p_R = \textstyle\sum_{i=1}^{n} \Delta p_i = \; & \Delta p_{S,f,h} \text{ (Solids friction horizontal) +} \\ & \Delta p_{S,f,v} \text{ (Solids friction vertical) +} \\ & \Delta p_{S,Hub} \text{ (Solids lifting) +} \\ & \Delta p_{S,acc} \text{ (Solids acceleration) +} \\ & \textstyle\sum \Delta p_{S.U} \text{ (Sum of solids deflections) +} \\ & \Delta p_F \text{ (Conveying gas)}\end{aligned} \quad (2)$$

For the individual terms, compare e.g. [1]:

2.1 Solids friction, horizontal straight pipe

For an incompressible conveyance in steady state, the following follows from a force balance at the solids analogous to single-phase flow:

$$\begin{aligned}-\Delta p_{S,f,h} &= \varepsilon_F \cdot \lambda_{S,h} \cdot \mu \cdot \frac{\Delta L_{R,h}}{D_R} \cdot \frac{\varrho_F}{2} \cdot u_F^2 \\ &\cong \lambda_{S,h} \cdot \mu \cdot \frac{\Delta L_{R,h}}{D_R} \cdot \frac{\varrho_F}{2} \cdot v_F^2\end{aligned} \quad (3)$$

The resistance coefficient $\lambda_{S,h}$ can theoretically be further broken down into two additive components: A particle/pipe wall component including particle/particle impact component $\lambda_{Stoß}$ and a wall friction component β_R due to sliding strands or plugs of solids supported on the pipe bottom or walls. In conveying tests, however, only the total value $\lambda_{S,h}$ can be determined by direct measurement.

The $\lambda_{S,h}$ values are measured by tests on short horizontal pipeline sections ΔL_R(-> incompressible flow) with correspondingly long upstream and downstream sections to ensure undisturbed flow. For the respective bulk material used it is possible to represent or correlate $\lambda_{S,h}$ as a function of the local Froude numbers $Fr_R = v_F^2 / (g \cdot D_R)$, with: g = gravitational acceleration (9,81 m/s²). Measurements show that $\lambda_{S,h}$ not only depends on the Fr_R-number, but, – especially in the case of fine-grained bulk solids – is also influenced by the loading μ and, if applicable, the size of the conveying pipe diameter D_R.There is a decrease in $\lambda_{S,h}$ at constant values of (μ, D_R) with increasing Fr_R number and, given a high $Fr_R, \lambda_{S,h}$ runs into a limit value, while it assumes smaller values at constant Fr_R with increasing loading μ as well as larger pipe diameter D_R.

Horizontal solids friction is usually the dominant contributor to the total pressure drop $|\Delta p_R|$ of a conveying pipeline.

2.2 Solids friction, vertical straight pipe

The calculation approach for $\Delta p_{S,f,v}$ is identical to that of equation (3):

$$-\Delta p_{S,f,v} = \varepsilon_F \cdot \lambda_{S,v} \cdot \mu \cdot \frac{\Delta L_{R,v}}{D_R} \cdot \frac{\varrho_F}{2} \cdot u_F^2 \cong \lambda_{S,v} \cdot \mu \cdot \frac{\Delta L_{R,v}}{D_R} \cdot \frac{\varrho_F}{2} \cdot v_F^2 \quad (4)$$

Since essentially only impact pressure drops and no sliding friction pressure drops occur in vertical piping, the vertical resistance coefficient $\lambda_{S,v}$ is generally smaller than the $\lambda_{S,h}$ of horizontal piping. This can be described by

$$\lambda_{S,v} \cong k_v \cdot \lambda_{S,h} \rightarrow \text{with: } 0.5 \lesssim k_v \leq 1.0 \quad (5)$$

In the dense phase range, i.e. at higher loadings μ, k_v values of the order of magnitude $k_v \cong 0.5$ are determined. With decreasing μ, i.e. transition into the entrained conveyance range, $k_v \rightarrow 1$. It follows from this that the friction pressure drop of a vertical conveying section can be determined in the same way as that of a horizontal section of the same length operated under identical boundary conditions. However, this applies with a resistance coefficient reduced by the factor k_v. The following equation therefore applies:

$$\Delta p_{S,f,v} \cong k_v \cdot \Delta p_{S,f,h} \quad (6)$$

$\lambda_{S,v}$ can be determined in the same way as $\lambda_{S,h}$, but the pressure drop measured over the pipe element under consideration must be reduced by the additional lifting pressure drop (-> see “Lifting the solids”).

2.3 Lifting the solids

The weight $(\Delta M_S \cdot g)$ of the mass of solids currently in the vertical conveying pipe $\Delta M_S = \dot{M}_S \cdot \overline{\Delta t_S} = \dot{M}_S \cdot \Delta L_{R,v} / \overline{u_S}$ (-> buoyancy is disregarded, since $\varrho_F \ll \varrho_S$) must be borne by the pressure differential force $(-\Delta p_{S,Hub} \cdot A_R)$, where A_R = pipe cross-sectional area. $\overline{\Delta t_S}$ is the mean residence time of the solids in the lifting section. From this, after some conversions with $\varepsilon_F = 1$ and $u_F \cong v_F$, it follows

$$-\Delta p_{S,Hub} \cong \frac{g}{\bar{C}_v} \cdot \frac{\dot{M}_S \cdot \Delta L_{R,v}}{A_R \cdot \bar{v}_F} = \frac{g}{\bar{C}_v} \cdot \mu \cdot \bar{\varrho}_F \cdot \Delta L_{R,v}$$
$$\rightarrow with{:}\ \bar{C}_v = \bar{u}_S / \bar{u}_F \cong \bar{u}_S / \bar{v}_F \quad (7)$$

The overlined values are mean values over the considered lifting height $\Delta L_{R,v}$. $\bar{C}_v$ describes the mean velocity ratio of solids and conveying gas, while $\bar{\varrho}_F$ represents the mean gas density.

2.4 Solids acceleration

The solids entering the conveying pipeline with the axial velocity $u_{S,in} = 0$ must first be accelerated to the stationary steady-state velocity $u_{S,stat} = C_{in} \cdot u_{F,in}$ at the start of the pipeline and then further along the conveying section to the respective current differential/slip velocity relative to the expanding conveying gas. Application of the principle of linear momentum

$$-\Delta p_{S,acc} \cdot A_R = \dot{M}_S \cdot \Delta u_S \quad (8)$$

leads with $u_{S,in} = 0$ and thus $\Delta u_S = u_{S,out} = C_{out} \cdot u_{F,out} \cong C_{out} \cdot v_{F,out}$ to the acceleration pressure drop of the solids along the entire conveying distance.

$$-\Delta p_{S,acc} \cong \dot{M}_S \cdot \frac{(C_{out} \cdot v_{F,out})}{A_R} \quad (9)$$

C_{out} is the velocity ratio of the solids and the conveying gas at the end of the pipeline.

2.5 Deflection of the solids

If the direction of flow of the gas-solids mixture is changed, the inertia of the solid particles and the centrifugal forces acting on them result in almost complete segregation of the gas and solids. The strand of solids pressed against the outside of the deflector (-> coarse-grained hard particles may also rebound) is slowed down by the increased wall friction to $u_{S,U,out}$ and must be accelerated back to its original steady-state speed $u_{S,U,in}$ downstream of the deflector. Application of the principle of linear momentum, equation (8), with

$$\Delta u_S = \left(u_{S,U,in} - u_{S,U,out}\right) = \left(\frac{\Delta u_{S,U}}{u_{S,U,in}}\right) \cdot u_{S,U,in}$$
$$= K_U \cdot C_{U,in} \cdot u_{F,U,in} \cong K_U \cdot C_{U,in} \cdot v_{F,U,in}$$

leads to

$$-\Delta p_{S,U} \cong K_U \cdot C_{U,in} \cdot \frac{\dot{M}_S \cdot v_{F,U,in}}{A_R}$$
$$= K_U \cdot C_{U,in} \cdot \mu \cdot \varrho_{F,U,in} \cdot v_{F,U,in}^2 \qquad (10)$$

Values indexed with "in" refer to the condition at the inlet to the deflector. The magnitude of the relative solids deceleration

$$K_U = \frac{\Delta u_{S,U}}{u_{S,U,in}} = \frac{u_{S,U,in} - u_{S,U,out}}{u_{S,U,in}}$$
$$= 1 - \frac{u_{S,U,out}}{u_{S,U,in}} \qquad (11)$$

depends, among other things, on the properties of the bulk material, the design of the deflector (-> pipe bend, deflector pot, T-bend, etc.) and its spatial arrangement (-> horizontal, upwards from the horizontal to the vertical, etc.). The K_U values of pipe bends arranged in the horizontal with arbitrary deflection angles α_U and a wall friction angle φ_W between the pipe wall and the bulk material can be calculated with

$$K_U = 1 - \exp\left(\frac{-\alpha_U}{180°} \cdot \pi \cdot \beta_R\right) \rightarrow with{:}\, \beta_R = tan\varphi_W \qquad (12)$$

When the solids are to be conveyed through pipe bends of ($\alpha_U = 90°$) arranged in this way, the following values can be used in equation (10):

- fine-grained bulk material: $\left(K_U \cdot C_{U,in}\right)_{90°RB} \cong 0.35$,
- coarse-grained bulk material: $\left(K_U \cdot C_{U,in}\right)_{90°RB} \cong 0.25$,
- vortex deflector: $\left(K_U \cdot C_{U,in}\right)_{Vortex} \cong 0.434$, fine-grained bulk material.

Further details and related references are given in [1].

2.6 Pressure drop of conveying gas

Since the conveying gas itself rubs against the pipe wall and flows through internals if applicable (-> resistance coefficient ξ_i), it has to be deflected and accelerated and lifted, resulting in an intrinsic pressure drop. The share of the gas in the total pressure drop of a pneumatic conveying system is negligible for denser conveying, i.e. loadings $\mu \geq 15$, but must be taken into account for lean phase conveying with lower loadings. In general, only the friction pressure drop $\Delta p_{F,f}$ and the acceleration pressure drop $\Delta p_{F,acc}$ of the gas are significant, i.e. the following applies with $u_F \cong v_F$

$$-\Delta p_F \cong -\left(\Delta p_{F,f} + \Delta p_{F,acc}\right)$$
$$\cong \bar{\lambda}_F \cdot \frac{L_R}{D_R} \cdot \frac{\bar{\varrho}_F}{2} \cdot \bar{v}_F^2 + \dot{M}_F \cdot \frac{\left(v_{F,out} - v_{F,in}\right)}{A_R} \qquad (13)$$

$\bar{\lambda}_F$, $\bar{\varrho}_F$ and $\bar{v}_F$ are suitably averaged characteristic values for the pipe friction resistance, the density and the velocity of the conveying gas. Equation (13) assumes that the conveying gas behaves as if the solids were not present. Of course, this is not the case: the presence of the bulk material particles changes the flow profile and thus the frictional pressure drop. However, this cannot be measured directly in conveying tests. Therefore, an undisturbed gas flow is generally used as an approximation.

To determine the pressure profile and thus the gas and solids velocity profile along the conveying distance L_R with the above-described Δp_R model, the distance is divided into correspondingly short ΔL_R sections and pipe elements, e.g. bends. These are generally calculated from the end of the pipeline, since the backpressure p_{out} is predetermined there and the associated gas velocity $v_{F,out}$ is relatively freely selectable, element by element in an incompressible manner using the equations shown above. The continuous progressions of (p, v_F, u_S) are thus approximated by "staircases". The smaller their steps, i.e. the ΔL_R length sections are selected, the more accurate the calculation, but also the greater the computational effort. Depending on the accuracy requirements, this section-by-section incompressible calculation also requires iterative calculation steps, e.g. to determine the local acceleration pressure drops of gas and solids.

3 Scale up model

3.1 Model building

The use of the standard model described in section 2 requires knowledge of the characteristics of the local resistance coefficients of the two-phase flow along the conveying pipeline. These are not available for the measurements described in section 1, which consider the entire pipeline. Here, only "global" characteristic values λ_{tot} of the respective piping are determined. In order to be able to use these results nevertheless, the following procedure is followed [2]:

From the measured λ_{tot}, which is dependent on the specific pipeline route, a mean solids-specific resistance coefficient $\bar{\lambda}_{S,h}$, cf. equation (3), can be calculated, which is independent of the pipeline route and thus transferable to other conveying systems. The gas wall friction coefficient $\bar{\lambda}_F$, equation (13), which can be scaled up in the same way, is determined using known clean gas approaches, e.g. the Blasius equation. To determine the characteristic values $(\bar{\lambda}_{S,h}, \bar{\lambda}_F)$, equations (1) and (2) are equalized under consideration of equations (3 – 13). Since conveyor pipelines always have vertical sections and deflectors distributed along them, the following is specified to simplify the calculations:

- The deflectors along the conveyor pipeline are converted into 90° pipe bends. Their number N_U results from the equation

Table 1 Comparison of $\bar{v}_F$ and $v_{F,geo}$ at $p_{out} = 1.0\ bar$

$\|\Delta p_R\|$ [bar]	$v_{F,in}$ [m/s]	$v_{F,out}$ [m/s]	$\bar{v}_F$ [m/s]	$v_{F,geo}$ [m/s]
1	10	20	13.86	14.14
1.5	10	25	15.27	15.81
2	5	15	8.24	8.66
2	10	30	16.48	17.32

Table 2 Characteristics of the two tested bulk solids

Characteristic	Unit	Sawdust	Limestone meal
Mean particle diameter $d_{S,50}$	$[\mu m]$	291	15
Inclination of the RRSB straights α_{RRSB}	[°]	56.2	34.2
Loose bulk density ϱ_{SS}	$[kg/m^3]$	320	1100
Shaken density ϱ_{SR}	$[kg/m^3]$	440	1680
Solids density ϱ_S	$[kg/m^3]$	1450	2720
Venting time $\Delta t(2\ kg)$	$[s]$	approx. 6	approx. 50
Geldart class	[-]	–	C
Particle shape	[-]	Flaky, fibrous	Rounded, spherical
Remarks	[-]	Extreme channel and rat hole formation during fluidization	Briquetted, agglomerated

$$N_U = \frac{\sum \alpha_U}{90°} \rightarrow with: \sum \alpha_U = \sum of\ all\ deflection\ angles \quad (14)$$

» All elevation sections and 90° pipe bends are flowed through with the average gas velocity $\bar{v}_F$, i.e. laid to the pipeline position of this average velocity $\bar{v}_F$.

» Calculation is based on a uniform average velocity ratio C of solids and conveying gas that is independent of the position along the conveying piping.

» The relative solids deceleration in the 90° pipe bends is calculated independently of their spatial arrangement and $\alpha_U = 90°$ using equation (12). The following therefore applies:

Table 3 Conveyor pipelines tested with sawdust

Pipeline data		Pipeline 1	Pipeline 2	Pipeline 3
Feeder type		Pressure vessel	Pressure vessel	Screw feeder
D_R	[mm]	54.5	24	82.5
L_R	[m]	63.4	46.6	140
$L_{R,v}$	[m]	7.8	7.9	7.8
$N_{U,90°}$	Eq. (14)	8	10	11.2

Table 4 Conveyor pipelines tested with limestone meal

Pipeline data		Pipeline 1	Pipeline 2	Pipeline 3
Feeder type		Pressure vessel	Pressure vessel	Pressure vessel
D_R	[mm]	54.5	82.5	82.5
L_R	[m]	159	159	62.2
$L_{R,v}$	[m]	5.8	5.6	6.2
$N_{U,90°}$	Eq. (14)	10	10	7

$$K_U = K_{U,90°} = 1 - exp\left(\frac{-\pi \cdot \beta_R}{2}\right) \quad (15)$$

Based on the above assumptions and taking into account that $(\bar{\varrho}_F \cdot \bar{v}_F) = \dot{M}_F\ /\ A_R$ can be set, the following relationship is obtained for an arbitrarily routed conveying pipeline:

$$\lambda_{tot} = \bar{\lambda}_{S,h} \cdot \left(\frac{L_{R,h}}{L_R} + k_v \cdot \frac{L_{R,v}}{L_R}\right) + \frac{\bar{\lambda}_F}{\mu} + 2 \cdot \frac{D_R}{L_R} \cdot \left(C \cdot \frac{v_{F,out}}{\bar{v}_F} + \frac{1}{\mu} \cdot \frac{v_{F,out} - v_{F,in}}{\bar{v}_F} + N_U \cdot K_U \cdot C\right) + \frac{2}{C} \cdot \frac{L_{R,v}}{L_R} \cdot \frac{g \cdot D_R}{\bar{v}_F^2} \quad (16)$$

The individual terms in equation (16) are dimensionless expressions/indicators, as required for a serious scale up. Reference values for $C, (K_U \cdot C), k_v$ etc. are given in section 2. $\bar{\lambda}_F$ values are estimated using Blasius‘ equation, which is valid for hydraulically smooth pipes in the Reynolds range Re_R that is relevant here:

$$\bar{\lambda}_F = \frac{0.3164}{Re_R^{1/4}} = 0.3164 \cdot \left(\frac{D_R \cdot \bar{v}_F \cdot \bar{\varrho}_F}{\eta_F}\right)^{-1/4} = 0.3164 \cdot \left(\frac{4}{\pi} \cdot \frac{\dot{M}_F}{D_R \cdot \eta_F}\right)^{-1/4} \quad (17)$$

η_F is the dynamic viscosity of the conveying gas at operating temperature. It is independent of pressure up to approx. 10 bar. Equation (16) shows that the influence of gas friction $\bar{\lambda}_F$ on the total resistance λ_{tot} decreases rapidly with increasing load $\mu (\rightarrow \bar{\lambda}_F\ /\ \mu)$.

If systematic tests with different operational settings are carried out on the same conveying pipeline with the same bulk material, $\bar{\lambda}_{S,h}$ can be calculated from equation (16) as a function of a Froude number $\bar{Fr}_R = \bar{v}_F^2/(g \cdot D_R)$ formed with the mean conveying gas velocity $\bar{v}_F$ and, if applicable, represented with (μ, D_R) as a parameter. These measurement curves form the basis for scaling up the pressure drop $|\Delta p_R|$ to other conveying pipelines.

Definition of the average conveying gas velocity $\bar{v}_F$: In order to determine a characteristic gas velocity $\bar{v}_F$, under the assumption of ideal gas behavior and a (in practice only approximate) linear decrease of the conveying pressure in the direction of conveyance, the integral mean velocity value $\bar{v}_F = \int_0^{L_R} v_F(L_X) \cdot dL_X / L_R$ is calculated to

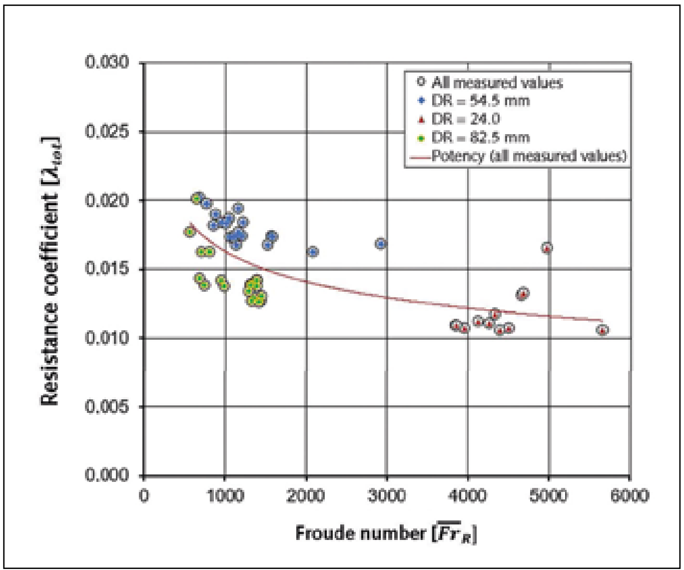
$$\bar{v}_F = \frac{v_{F,out} \cdot p_{out}}{|\Delta p_R|} \cdot ln\left(\frac{v_{F,out}}{v_{F,in}}\right) \quad (18)$$

Comparisons with the geometric mean value from the gas inlet and outlet velocities

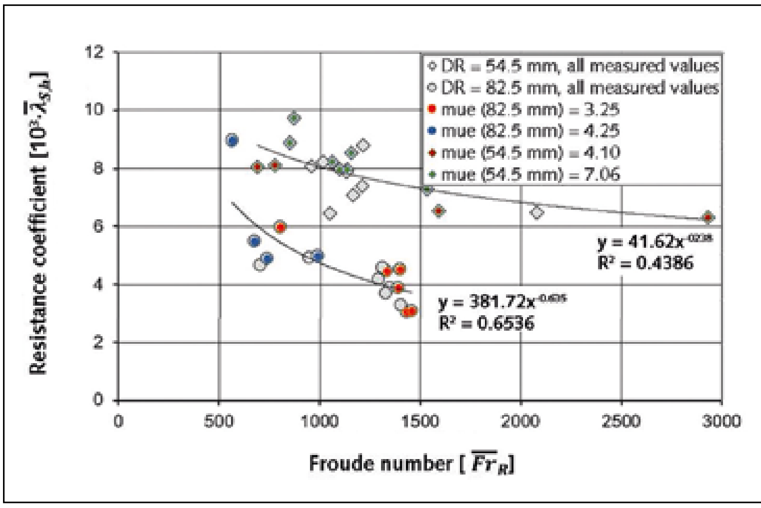
$$v_{F,geo} = \sqrt{v_{F,in} \cdot v_{F,out}} \quad (19)$$

only show a slight difference from $\bar{v}_F$, cf. Table 1.

In consideration of the real pressure curves and for simpler manageability, the geometrically averaged gas velocity is therefore used as refer-



1 Influence of the particular pipeline layout on $\lambda_{tot}(\overline{Fr}_R)$, sawdust tests



2 Dependency $\bar{\lambda}_{S,h}(\overline{Fr}_R)$ of the sawdust tests, $D_R = 54.5\ mm/82.5\ mm$

ence velocity in the following considerations, i.e. $\bar{v}_F = v_{F,geo}$ is set.

3.2 Measurement results

To illustrate the described procedure, the measurement results for λ_{tot} determined with two very different bulk solids and the $\bar{\lambda}_{S,h}$ values back-calculated from them are shown below as examples. The two bulk materials are, firstly, a relatively fine-grained sawdust [2] and, secondly, a very finely ground limestone meal [3]. Both were conveyed under systematically varied operating conditions through pipes with different routes, diameters D_R and lengths L_R.

Table 2 summarizes some characteristics of the two bulk materials. The sawdust was conveyed through the three conveying pipelines shown in Table 3, while the limestone meal was measured during conveyance through the pipelines described in Table 4. Using equation (16) converted according to $\bar{\lambda}_{S,h}$

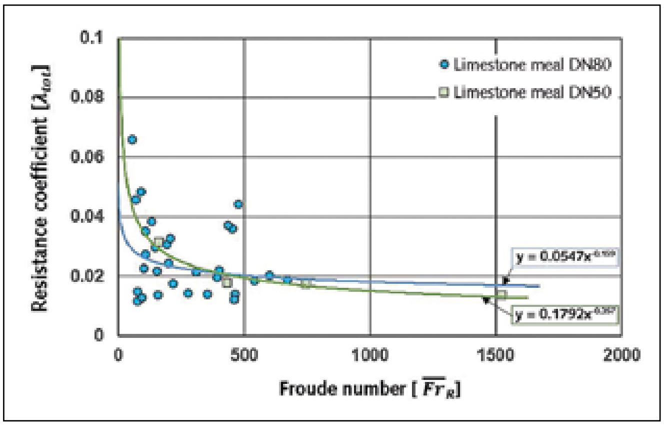
$$\bar{\lambda}_{S,h} = \frac{\lambda_{tot} - \frac{\bar{\lambda}_F}{\mu} - 2 \cdot \frac{D_R}{L_R} \cdot \left(C \cdot \frac{v_{F,out}}{\bar{v}_F} + \frac{1}{\mu} \cdot \frac{v_{F,out} - v_{F,in}}{\bar{v}_F} + N_U \cdot K_U \cdot C \right) - \frac{2}{C} \cdot \frac{L_{R,v}}{L_R} \cdot \frac{1}{\overline{Fr}_R}}{\left(\frac{L_{R,h}}{L_R} + k_v \cdot \frac{L_{R,v}}{L_R} \right)} \quad (16a)$$

the corresponding solid-specific $\bar{\lambda}_{S,h}$ values can be calculated from the measured λ_{tot} data. In accordance with the given experimental conditions, the following operating variables were used to evaluate equation (16a): Conveying gas = nitrogen (-> sawdust) or ambient air (-> limestone meal), $T_S = 20°C$, $p_{out} = 1.0\ bar$, $C = 0.70$, $k_v = 1.00$, $(K_U \cdot C) = 0.35, \eta_F$ corresponding to the type of gas and the temperature.

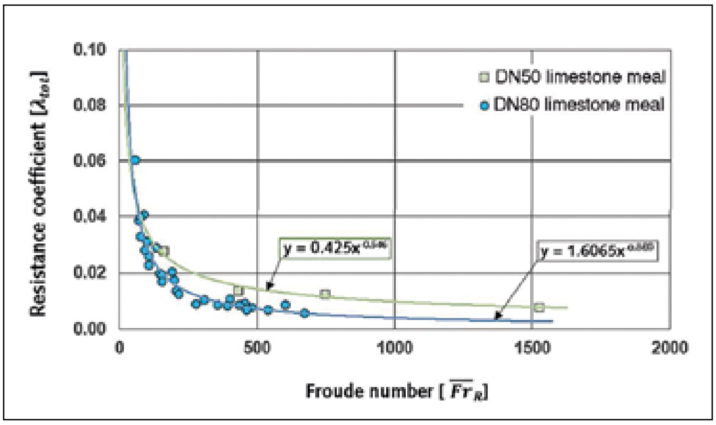
Figure 1 shows the dependencies measured with sawdust in the three tested pipelines $\lambda_{tot}(\overline{Fr}_R = \bar{v}_F^2 / (g \cdot D_R))$. The pressure differences at the conveyor sections were varied in the range $\Delta p_R \cong (0.12 \cdots 1.35) bar$ and the loadings were varied in the range of $\mu \cong (1.0 \cdots 9.0) kg_s/kg_F$. In an operation-reliable manner, it was only possible to realize entrained phase conveying with gas velocities of $v_{F,in} \gtrsim 17.0\ m/s$ at the beginning of the pipeline [3].

The greatly differing pipeline layouts of the three conveying systems result in three different and separate groups of measured values. Based on the explanations in section 2, this was also to be expected.

In Figure 2, the scale-up capable horizontal resistance coefficients $\bar{\lambda}_{S,h}$ calculated with equation (16a) from the measured λ_{tot} values are plotted as a function of the geometrically averaged Froude number $\overline{Fr}_R$. In order to clarify details, only the area $\overline{Fr}_R \leq 3000$ is shown. It can be seen that the piping diameter D_R at a given $\overline{Fr}_R$ has an influence on the magnitude of the resistance coefficient $\bar{\lambda}_{S,h}$: The larger the D_R, the smaller the $\bar{\lambda}_{S,h}$. It should be noted here that in Figure 2 the ordinate is smaller by a factor of 10 than that of Figure 1, i.e. the absolute $\bar{\lambda}_{S,h}$ deviations for variable D_R are relatively small compared to Figure 1. However, as extensive control calculations show, they are real. This effect can be explained by the fact that the ratio of pipe circumference U_R to pipe cross-sectional area A_R increases with decreasing pipe diameter A_R, i.e. under comparable operating conditions ($\rightarrow e.g. \overline{Fr}_R, \mu = const.$) a smaller D_R will result in more frequent particle/pipe wall contacts, thus causing a greater pressure drop than would happen in a pipe with a larger diameter D_R. The reduction in the ratio of the measured $\bar{\lambda}_{S,h}(82.5\ mm) / \bar{\lambda}_{S,h}(54.5\ mm)$ values with increasing pipe diameter D_R depends on the current $\overline{Fr}_R$ number.



3 **Influence of the particular pipeline layout on $\lambda_{tot}(\overline{Fr}_R)$, limestone meal tests**



4 **Dependency $\bar{\lambda}_{S,h}(\overline{Fr}_R)$ of the limestone meal tests,** $D_R = 54.5\ mm/82.5\ mm$

No systematic influence of the loading μ on $\bar{\lambda}_{S,h}$ is evident from Figure 2. However, it can be seen that both diameter curves rise very steeply in the area $\overline{Fr}_R \rightarrow 500$ with the higher realized loadings μ (-> is not shown by the compensation curves). This indicates an approach to the physical conveying limit, i.e. $\overline{Fr}_R - numbers \lesssim 500$ should not be implemented in industrial systems.

Figure 3 shows the dependencies $\lambda_{tot}(\overline{Fr}_R)$ determined with the limestone meal characterized in Table 2 in the pipelines described in Table 3. The pressure differences in the conveying pipelines were varied in the range of $\Delta p_R \cong (1.04 \cdots 2.06) bar$ and the corresponding loadings of $\mu \cong (13.4 \cdots 125.5) kg_S/kg_F$. Initial conveying gas velocities down to $v_{F,in} \cong 4.5\ m\ /\ s$ were possible with operational reliability. Consequently, conveying was possible in both the lean flow and dense flow ranges [3]. A classification into different μ ranges was not made in Figure 3. The three relatively high measured values of λ_{tot} in the range $\overline{Fr}_R \cong 490$ were determined during conveyances with the smallest loads μ.

5 **Claudius Peters Fluidcon system for pneumatic conveying**

Figure 4 shows the dependencies of the resistance coefficients $\bar{\lambda}_{S,h}(\overline{Fr}_R)$ of the limestone meal calculated with equation (16a) from the measured λ_{tot} values. The $\bar{\lambda}_{S,h}$ values of the different pipe lengths with the pipe diameter $D_R = 82.5\ mm$ lie on a common curve. As in the case of sawdust, the coefficient of resistance $\bar{\lambda}_{S,h}$ decreases with increasing pipe diameter D_R for a given $\overline{Fr}_R$.

The magnitude of the change in $\bar{\lambda}_{S,h}$ values with the pipe diameter D_R is again dependent here on the current $\overline{Fr}_R$ number.

3.3 Scale-up to other conveying pipelines

The following procedure is proposed for transferring the pressure drop measurements determined in a test system, index "V", to a planned industrial system, index "B", with a specified solids mass flow rate $\dot{M}_{S,B}$ and back pressure $p_{R,out,B}$ at the end of the pipeline:

1. Calculation of the mean conveying gas velocities $\bar{v}_{F,V}$ of the individual tests by means of equation (19).
2. Calculation of the $\lambda_{tot,V}$ values of the individual tests with equation (1).
3. Determination of the $\bar{\lambda}_F$ values of the individual tests by means of equation (17).
4. Calculation of the $\bar{\lambda}_{S,h}$ values of the individual tests from equation (16a).
5. Representation of the $\bar{\lambda}_{S,h}$ test values as a function $\bar{\lambda}_{S,h}(\overline{Fr}_R, \mu, D_R)$ with

$$\overline{Fr}_R = \frac{\bar{v}^2_{F,V}}{g \cdot D_{R,V}} = \frac{v_{F,in,V} \cdot v_{F,out,V}}{g \cdot D_{R,V}}\ and\ \mu = \frac{\dot{M}_{S,V}}{\dot{M}_{F,V}} \quad (20a,b)$$

» $\bar{\lambda}_{S,h}(\overline{Fr}_R, \mu, D_R)$ can be represented over $\overline{Fr}_R$ in a diagram using $\mu\ resp.\ D_R$ as parameters or correlated by a suitable function. The function $\bar{\lambda}_{S,h}(\overline{Fr}_R, \mu, D_R)$ is the basis for scaling up to a planned industrial system. $f(\bar{\lambda}_{S,h}) = f(\bar{\lambda}_{S,h,V}) = f(\bar{\lambda}_{S,h,B})$ and $f(\bar{\lambda}_F) = f(\bar{\lambda}_{F,V}) = f(\bar{\lambda}_{F,B})$ apply for the functional dependencies of $\bar{\lambda}_{S,h}$ and $\bar{\lambda}_F$. Thus, they are applicable to both the test system and the industrial system.

6 **Modern test facility – 5 km long pneumatic conveying pipe**

For the pressure drop calculation of the respective industrial system, the following steps are then required:

6. Specification/estimation of a conveying pipeline diameter $D_{R,B}$.
7. Determination of a suitable initial conveying gas velocity $v_{F,in,B}$ (or final conveying gas velocity $v_{F,out,B}$).
8. Estimation of the pressure drop $\Delta p_{R,B}$ to be expected in the industrial system.
9. Calculation of the associated final conveying gas velocity $v_{F,out,B}$ (or initial conveying gas velocity $v_{F,in,B}$) and the resulting average gas velocity $\bar{v}_{F,B}$ using $\Delta p_{R,B}$.
10. Determination of the required gas mass flow rate $\dot{M}_{F,B}$ and loading μ_B at the given solids mass flow rate $\dot{M}_{S,B}$.
11. Calculation of the $\overline{Fr}_R$ number analogous to equation (20a) or respectively (19) and determination of the $\bar{\lambda}_{S,h}$ value of the industrial system from the $\bar{\lambda}_{S,h}(\overline{Fr}_R, \mu, D_R)$ diagram determined in the test system or respectively from the correlation equation.
12. Determination of the $\bar{\lambda}_F$ value of the industrial system by means of equation (17).
13. Back-calculation of the $\lambda_{tot,B}$ value of the industrial system by means of equation (16) with $\bar{\lambda}_{S,h}$ from work step 11, $\bar{\lambda}_F$ from work step 12 and the operating data of the planned conveying system.
14. Calculation of a corrected pressure drop $\Delta p_{R,B,kor}$ with equation (1) and the $\lambda_{tot,B}$ determined in work-step 13.
15. Comparison of the estimated value $\Delta p_{R,B}$ with $\Delta p_{R,B,kor}$: If $|\Delta p_{R,B} - \Delta p_{R,B,kor}| > \delta$, where δ is a suitably selected accuracy limit, then $\Delta p_{R,B} = \Delta p_{R,B,kor}$ is set and you jump back to step 9. This iterative loop is repeated until $|\Delta p_{R,B} - \Delta p_{R,B,kor}| \leq \delta$. Practical calculations show that the iteration leads to a stable final value for $\Delta p_{R,B}$ after approx. 3 runs.

The pressure drop calculation of a pneumatic conveying pipeline by means of the λ_S method generally requires an iterative procedure.

The dependence $\bar{\lambda}_{S,h} \propto 1/D_R^m$ determined on the basis of the sawdust and limestone meal measurements is also observed for other bulk solids [1]. If $D_{R,B} > D_{R,V}$ is implemented for such solids, then the pressure drop of the industrial system is on the "safe" side, i.e., it is bigger than necessary, and vice versa.

Table 5 Iteration process, conveyor pipeline $D_R = 127.1\ mm$, unstaggered

Circulation	[No.]	1	2	3
$\|\Delta p_{R,B}\|$	[bar]	0.4	0.281	0.373
$v_{F,in,B}$	[m/s]	24.97	26.1	**25.21**
$v_{F,out,B}$	[m/s]	34.95	33.43	**34.62**
$\dot{M}_{F,B}$	[kg/h]	1915.6	1832.3	**1897.4**
μ_B	[kg$_S$/kg$_F$]	2.98	3.11	**3**
$\bar{\lambda}_{S,h}$	[1]	$6.0 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$	**$6.0 \cdot 10^{-3}$**
$\bar{\lambda}_F$	[1]	0.0136	0.0138	**0.0136**
$\lambda_{tot,B}$	[1]	0.0129	0.0176	**0.0174**
$\|\Delta p_{R,B}\|_{kor}$	[bar]	0.281	0.373	**0.378**

3.4 Application example

Design data:

- The sawdust tested above is to be used for feeding a burner system, i.e. a low-pulsation, continuous bulk material feed is required,
- Pressure at the end of the conveying pipeline: $p_{out,B} = 1.00\ bar$,
- Design temperature: $T_B = 20\ °C$,
- Bulk material: sawdust, according to Table 2,
- Conveying gas: ambient air, density $\varrho_{Luft}(20\ °C, 1.0\ bar) = \varrho_{F,out,B} = 1.20\ kg\ /\ m^3$ dynamic viscosity $\eta_{Luft}(20\ °C, 1.0\ bar) = \eta_{F,T_B} = 18.24 \cdot 10^{-6} Pa \cdot s$,
- Solids mass flow rate: $\dot{M}_{S,B} = 5.70\ t/h$,
- Conveying distance: $L_{R,B} = 150\ m$,
- Included vertical height: $\Delta L_{R,v,B} = 19\ m$,
- Number of deflectors converted to 90° bends, cf. equation (14) : $N_U = 8$,
- Specified conveying pipeline diameter: $D_{R,B} = 127.1\ mm$ (-> pipe $\emptyset$ $139.7\ mm\ x\ 6.3\ mm$).

Calculation example of the industrial system:

The further procedure is in accordance with section 3.3, whereby the $\bar{\lambda}_{S,h}$ test data are already available (-> work steps 1 - 5) and can be taken from Figure 2, $D_R = 82.5\ mm$.

- Lower conveying/operating limit: As the lower limit for stable and low-pulsation conveying operation (-> the latter can be checked by means of the pressure measuring strips of the corresponding conveying tests and is required for a burner system), the Froude number $\overline{Fr}_{R,min} \gtrsim 500$ stated in section 3.2, comments on Figure 2, is used and a safety margin is added.

  The selected:

$$\overline{Fr}_{R,B} = \frac{\bar{v}_F^2}{g \cdot D_R} = 700$$

with

$$v_{F,out} = v_{F,in} \cdot \frac{p_{in}}{p_{out}}$$

$$\overline{Fr}_{R,B} = \frac{v_{F,in} \cdot v_{F,out}}{g \cdot D_R} = \frac{v_{F,in}^2}{g \cdot D_R} \cdot \frac{p_{in}}{p_{out}}$$

produces the following as a general rule for the conveying gas velocity $v_{F,in}$ at the beginning of the pipeline

$$v_{F,in} = \sqrt{g \cdot D_R \cdot \overline{Fr}_{R,B} \cdot \frac{p_{out}}{p_{in}}} \quad (21)$$

The following therefore applies to the system under consideration:

$$v_{F,in,B} = \sqrt{g \cdot D_{R,B} \cdot \overline{Fr}_{R,B} \cdot \frac{p_{out,B}}{p_{in,B}}}$$

$$= \sqrt{9.81 \frac{m}{s^2} \cdot 0.1271\ m \cdot 700 \cdot \frac{p_{out,B}}{p_{in,B}}}$$

$$= 29.54 \frac{m}{s} \cdot \sqrt{\frac{p_{out,B}}{p_{in,B}}} \quad (21a)$$

- Pressure drop calculation: $C = 0.70, k_v = 1.00, (K_U \cdot C) = 0.35$ are used as fixed values in equation (16) for the calculation of the total resistance coefficient $\lambda_{tot,B}$ of the industrial system. It follows that:

$$\left(\frac{L_{R,h,B}}{L_{R,B}} + k_v \cdot \frac{L_{R,v,B}}{L_{R,B}}\right) = 1.0$$

$$\frac{2}{C} \cdot \frac{L_{R,v,B}}{L_{R,B}} = \frac{2}{0.70} \cdot \frac{19\ m}{150\ m} = 0.3619,$$

$$2 \cdot \frac{D_{R,B}}{L_{R,B}} = 2 \cdot \frac{0.1271\ m}{150\ m} = 1.6947 \cdot 10^{-3}$$

$$N_U \cdot K_U \cdot C = 8 \cdot 0.35 = 2.80$$

In the following, the design steps **7** ff of section 3.3 are implemented.

Estimated pressure drop: $|\Delta p_{R,B}| = 0.40\ bar$, with $p_{out,B} = 1.00\ bar$ produces $p_{in,B} = 1.40\ bar$.

Conveying gas inlet velocity, equation (21a):

$$v_{F,in,B} = 29.54 \frac{m}{s} \cdot \sqrt{\frac{p_{out,B}}{p_{in,B}}} = 29.54 \frac{m}{s} \cdot \sqrt{\frac{1.00\ bar}{1.40\ bar}}$$

$$= 24.97 \frac{m}{s}$$

Conveying gas outlet velocity

$$v_{F,out,B} = v_{F,in,B} \cdot \frac{p_{in,B}}{p_{out,B}} = 24.97 \frac{m}{s} \cdot \frac{1.40\ bar}{1.00\ bar}$$

$$= 34.95 \frac{m}{s}$$

Geometrically averaged gas velocity, equation (19):

$$\bar{v}_{F,B} = \sqrt{v_{F,in,B} \cdot v_{F,out,B}} = \sqrt{24.97 \frac{m}{s} \cdot 34.95 \frac{m}{s}}$$

$$= 29.54 \frac{m}{s}$$

Conveying gas mass flow:

$$\dot{M}_{F,B} = \frac{\pi}{4} \cdot D_{R,B}^2 \cdot v_{F,Out,B} \cdot \varrho_{F,out,B}$$

$$= \frac{\pi}{4} \cdot (0.1271\, m)^2 \cdot 34.95 \frac{m}{s} \cdot 1.20 \frac{kg}{m^3} \cdot 3600 \frac{s}{h}$$

$$= 1915.6 \frac{kg}{h}$$

Loading:

$$\mu_B = \frac{\dot{M}_{S,B}}{\dot{M}_{F,B}} = \frac{5.70 \frac{t}{h} \cdot 1000 \frac{kg}{t}}{1915.6 \frac{kg}{h}} = 2.98 \frac{kg_S}{kg_F}$$

Determination of the solids resistance coefficient $\bar{\lambda}_{S,h}$ for the operating conditions , $\overline{Fr}_{R,B} = 700$, $\mu_B = 2.98$ and $D_R = 82.5\, mm$ from Figure 2:

$$\bar{\lambda}_{S,h} \cong 6.0 \cdot 10^{-3}$$

Calculation of the resistance coefficient $\bar{\lambda}_F$ of the conveying gas, equation (17):

$$\bar{\lambda}_F = 0.3164 \cdot \left(\frac{4}{\pi} \cdot \frac{\dot{M}_{F,B}}{D_{R,B} \cdot \eta_F}\right)^{-\frac{1}{4}}$$

$$= 0.3164 \cdot \left(\frac{4}{\pi} \cdot \frac{1915.6 \frac{kg}{h}}{3600 \frac{s}{h} \cdot 0.1271 m \cdot 18.24 \cdot 10^{-6} Pa \cdot s}\right)^{-\frac{1}{4}}$$

$$= 0.0136$$

Back-calculation of the total resistance coefficient $\bar{\lambda}_{tot,B}$ using equation (16):

$$\bar{\lambda}_{tot,B} = \bar{\lambda}_{S,h} \cdot \left(\frac{L_{R,h,B}}{L_{R,B}} + k_v \cdot \frac{L_{R,v,B}}{L_{R,B}}\right) + \frac{\bar{\lambda}_F}{\mu_B} + 2 \cdot \frac{D_{R,B}}{L_{R,B}} \cdot \left(C \cdot \frac{v_{F,out,B}}{\bar{v}_{F,B}} + \frac{1}{\mu_B} \cdot \frac{v_{F,out,B} - v_{F,in,B}}{\bar{v}_{F,B}} + N_U \cdot K_U \cdot C\right) + \frac{2}{C} \cdot \frac{L_{R,v,B}}{L_{R,B}} \cdot \frac{g \cdot D_{R,B}}{\bar{v}_{F,B}^2}$$

$$= 6.0 \cdot 10^{-3} \cdot (1) + \frac{0.0136}{2.98} + 1.6947 \cdot 10^{-3} \cdot \left(0.70 \cdot \frac{34.95 \frac{m}{s}}{29.54 \frac{m}{s}} + \frac{1}{2.98} \cdot \frac{34.95 \frac{m}{s} - 24.97 \frac{m}{s}}{29.54 \frac{m}{s}} + 2.80\right) + \frac{0.3619}{700} = 0.0129$$

Using equation (1), a corrected pressure drop $|\Delta p_{R,B}|_{kor}$ of the industrial system is calculated. Equation (1) can be transformed as follows by inserting $(\bar{\varrho}_F \cdot \bar{v}_F) = \dot{M}_F / A_R$ and $\mu = \dot{M}_S / \dot{M}_F$:

$$|\Delta p_{R,B}|_{kor} = \frac{2}{\pi} \cdot \lambda_{tot,B} \cdot \dot{M}_{S,B} \cdot \frac{L_{R,B}}{D_{R,B}^3} \cdot \bar{v}_{F,B} \qquad (1a)$$

$$|\Delta p_{R,B}|_{kor} = \frac{2}{\pi} \cdot 0.0129 \cdot \frac{5.70 \frac{t}{h} \cdot 1000 \frac{kg}{t}}{3600\, s} \cdot \frac{150\, m}{(0.1271\, m)^3} \cdot 29.54 \frac{m}{s} = 28061.2\, Pa$$

$$= 0.281\, bar$$

Repeated insertion of the respectively determined $|\Delta p_{R,B}|_{kor}$ value as a new estimated/starting value $|\Delta p_{R,B}|$ in the above calculation process leads to the results of the iteration summarized in Table 5. $\overline{Fr}_{R,B} = 700 = konst.$ applies here for all iterations. After three iteration steps the relative error is $(|\Delta p_R|_{kor} - |\Delta p_R|) / |\Delta p_R| \cong 1.34\%$.

The iteration is then stopped, since no more precise values can be read from the existing $\bar{\lambda}_{S,h}$ diagram, Figure 2, anyway. All data relevant for the plant design are shown in bold print.

The described calculation can be accelerated by suitable iteration methods, e.g. selection of $|\Delta p_{R,B}|_{neu} = \left(|\Delta p_{R,B}|_{alt} + |\Delta p_{R,B}|_{kor}\right) / 2$. Further information on the design of pneumatic conveying systems can be found inter alia in [1, 4].

» Comments:

Since the $\bar{\lambda}_{S,h}$ reference curve used for the above calculations was that of pipeline diameter $D_R = 82.5\, mm$, although the sawdust test measurements indicate a reduction in the $\bar{\lambda}_{S,h}$ value with increasing conveying pipeline diameter, the pressure drop calculation is on the “safe” side. Further investigations are required in this respect.

The detailed results of the calculations performed above show that the deflection pressure drops $(N_U \cdot K_U \cdot C)$ in equation (16) cause the greater part of the additional pressure drops of the industrial system. This should be true for all predominantly horizontal conveying pipelines. For higher loadings μ, the influence of gas friction and the acceleration component of the gas can generally be neglected. This must be decided on a case-by-case basis.

REFERENCES

[1] Hilgraf, P.: Pneumatische Förderung – Grundlagen Auslegung und Betrieb von Anlagen. Springer Vieweg, Berlin (2019)

[2] Hilgraf, P.: Scale-up der Ergebnisse pneumatischer Förderversuche auf Betriebsanlagen. Interner Entwicklungsbericht der Claudius Peters Projects GmbH, Buxtehude/Germany, Mai/Juni 2022, unveröffentlicht

[3] Hirschfeld, K.: Erstellung eines Auswertemodells zum detaillierten Scale-Up für die Auslegung einer Realanlage und die Untersuchung eines Schüttgutes auf die Feststoffgeschwindigkeiten in der pneumatischen Förderung. Bachelorarbeit, Hochschule für angewandte Wissenschaften Hamburg (HAW), Studiengang Maschinenbau und Produktion 2022, unveröffentlicht

[4] Hilgraf, P.: Projektierung und Auslegung von Schüttgutanlagen – Darstellung anhand von Praxisbeispielen. Springer Vieweg, Berlin (2022)